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Performance Comparison of LMS and RLS in Adaptive Equalization for Different Channel Conditions

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Abstract:-The Least Mean Squares (LMS) algorithm is an important member of the family of stochastic gradient algorithms. A significant feature of the LMS algorithm is its simplicity. The recursive least squares (RLS) algorithm recursively finds the filter coefficients for minimizing linear least squares cost function. Adaptive equalization is capable of tracking a slowly time-varying channel response. For given channel models the LMS and RLS algorithms' performance is simulated by using MATLAB and it is clearly obtained that the RLS algorithm is better than LMS algorithm because it provides faster and better convergence. It seems more stable than LMS algorithm.

Index Terms—LMS, RLS, channel equalization.

I. INTRODUCTION

The LMS algorithm is an important member of the family of stochastic gradient algorithms. The term „stochastic gradient" is intended to distinguish the LMS algorithm from the method of steepest descent which uses a deterministic gradient in a recursive computation of the Wiener filter for stochastic inputs [1]. Significant features of the LMS algorithm are its simplicity and robust performance against different channel conditions [4]. However, it has slow convergence and does not have robustness for high channel changes [8]. The LMS algorithm is a linear adaptive filtering algorithm, which in general consists of two basic processes [9]:

A filtering process, which involves computing the output of a linear filter in response to an input signal and generating an estimation error by comparing this output with a desired response. An adaptive process which involves the automatic adjustment of the parameters of the filter in accordance with the estimation error. The LMS algorithm performs the following operations to update the coefficients of an adaptive filter:

1- Calculates the output signal y(n) from the adaptive filter.
2- Calculates the error signal e(n) by using the following equation:

$$n \; = \; n \; - y(n)$$

3.Updates the filter coefficients by using the following equation:

$$w \;\; n+1 \;\; = w \;\; n \;\; + \qquad n \;\; u(n)$$

The RLS algorithm recursively finds the filter coefficients that minimize a weighted linear least squares cost function relating to the input signals. It has fast convergence and it considers channel changes but it comprises complex processes and it is costly in engineering point of view [13]. In contrast to other algorithms such as LMS that aim to reduce the mean square error. In the derivation of the RLS, the input signals are considered deterministic on the other hand in LMS they are considered stochastic [15]. All necessary equations for RLS algorithm are given below:

1- $n \;=\; n \;-u \; n\;^{T}w \;\; n-1$
2- $k \;\; n \;=\; \dfrac{^{-1}P \;\; n-1 \;\; u(n)}{1+ \;\; ^{-1}u^{T} \;\; n \;\; P \;\; n-1 \;\; u(n)}$
3- $\alpha \;\; n \;=\; n \;-u \;\; n\;^{T}w(n-1)$
4- $w \;\; n \;=w \;\; n-1 \;\; +k \;\; n \;\; \alpha(n)$
5- $P \;\; n \;=\; ^{-1}P \;\; n-1 \;\; - \;\; ^{-1}k \;\; n \;\; u^{T} \;\; n \;\; P(n-1)$

Equalization, capable of tracking a slowly time-varying channel response [5], is known as adaptive equalization. It can be implemented to perform tap-weight adjustments periodically or continually. Periodic adjustments are accomplished by periodically transmitting a preamble or short training sequence of digital data known by the receiver. Continual adjustment is accomplished by replacing the known training sequence with a sequence of data symbols estimated from the equalizer output and treated as known data. When performed continually and automatically in this way, the adaptive procedure is referred to as decision directed. If the probability of error exceeds one percent, the decision directed equalizer might not converge. A common solution to this problem is to initialize the equalizer with an alternate process, such as a preamble to provide good channel-error performance, and then switch to decision-directed mode [19]. To obtain a stable solution to the filter weights, it is necessary that the data be averaged to obtain the

stable signal statistic, or the noisy solution obtained from the noisy data must be averaged. The most robust algorithm that average noisy solution is the LMS algorithm. Each iteration of this algorithm uses a noisy estimate of the error gradient to adjust the weights in the direction to decrease the average mean-square error [20].

II. PERFORMANCE EVOLUTION

For the given channel models, The LMS and RLS algorithm results are obtained. Simulations are achieved for LMS algorithm with different values of step size parameter „ ″. The optimum value of delay which minimizes the mean square error at the equalizer output is determined. According to the optimum delay, the learning curve of the equalizer for different step-size parameters and iteration numbers are plotted.

For given channel models the RLS algorithm with the different values of regularization parameter ( ) is implemented. The optimum value of delay which minimizes the mean square error at the equalizer output is also determined for RLS algorithm. The learning curve of the equalizer for different regularization parameters and iteration numbers is achieved for optimum delay which is determined. The input signal is random bipolar sequence ($[-1,1]$). The noise is Additive White Gaussian Noise (AWGN) [2] with mean 0 and variance is 0.001. The channel models are determined below:

$$h_1 = [0.25 \quad 1 - 0.25]$$
$$h_2 = [0.3887 \;\; 1 \quad 0.3887]$$
$$h_3 = [0.0125 \;\; 0.5 \;\; 0.9875]$$

For channel 1, the impulse response is symmetric around $n = 1$ so this channel introduces group delay of 1 unit. Second channel and third channel is obtained from linear channel models with cosines and sinus functions which are shown below [3].

$$h\, n \;=\; \frac{1}{2}\; 1 + \cos\left(\frac{2\pi}{W}(n-2)\right)$$

III) RESULTS AND ANALYSIS

For different simulation parameters the LMS and RLS results are obtained separately in terms of three channel conditions. For determining an effects of the simulation parameters, different step-size, iteration numbers, delay and number of taps are taken into consideration. Figure 1,2,3 and 4 show the simulation results of LMS algorithm. In figures, red line corresponds first channel, green line is for second channel and blue line is for third channel. Table 1 shows the parameter values for Figures 1 to 4 for LMS algorithm.

Fig.	Simulation Parameters				
No	Trial		# of iterations	delay	taps
1	100	0.001	2000	11	21
2	100	0.1	2000	11	21
3	100	0.000001	2000	11	21
4	100	0.001	10000	11	21

Table 1: LMS Simulation Parameters

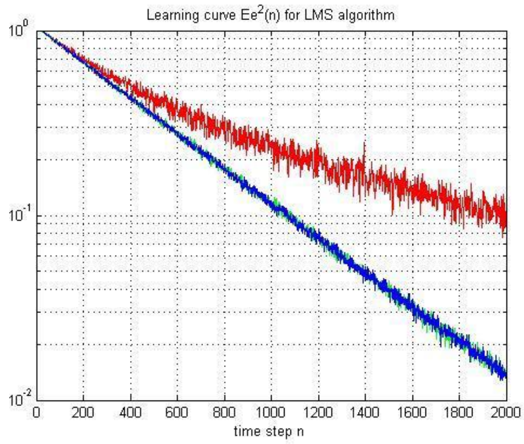

Figure 1

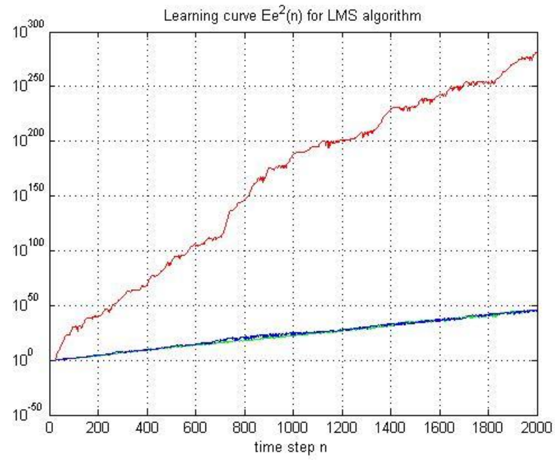

Figure 2

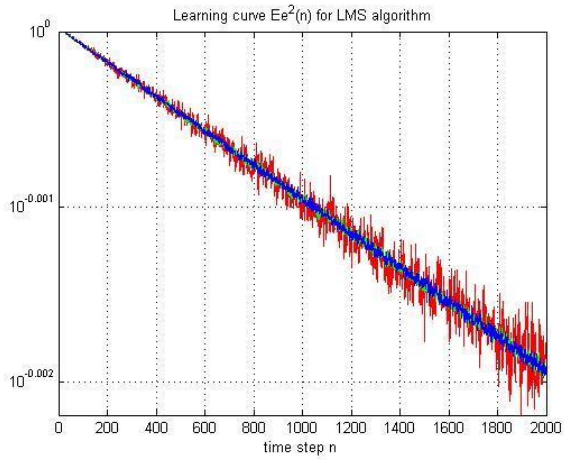

Figure 3

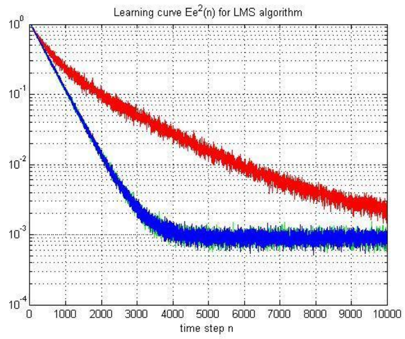

Figure 4

Figures 5-8 show the simulation results of RLS algorithm and simulation parameters for these figures are shown in Table 2.

Fig.	Simulation Parameters				
No	Trial		# of iterations	delay	taps
1	100	0.005	2000	11	21
2	100	0.5	2000	11	21
3	100	0.000005	2000	11	21
4	100	0.005	10000	11	21

Table 2: RLS Simulation Parameters

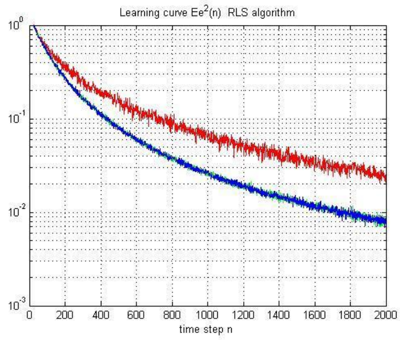

Figure 5

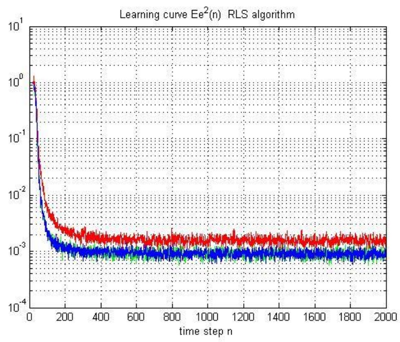

Figure 6

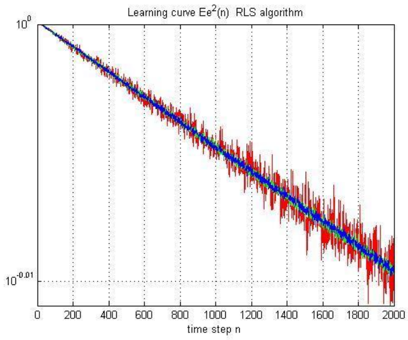

Figure 7

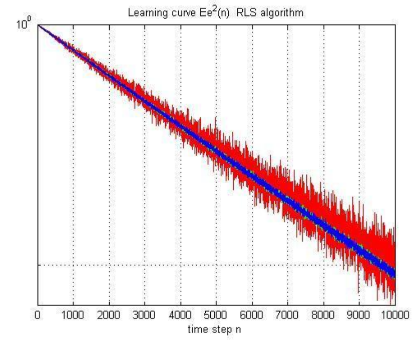

Figure 8

It is clearly seen from the results that the channel forms, step-size parameter and iteration number affects the LMS results of simulation. If step–size parameter is very high, the LMS algorithm does not provide good convergence as seen from Figure 2. If the step-size parameter is very low, as seen from the Figure 3, the LMS algorithm does not have good convergence too. The best convergence is taken when the step–size parameter is optimum and iteration number is very high (10000) as seen from Figure 4. If the step–size parameter is chosen low it causes very small changes on weight vector. If the step–size parameter is chosen high, it may cause instability. So the key point in LMS algorithm is choosing step-size parameter properly.

The channel forms, regularization parameter and iteration number affects the RLS results of simulation. If the regularization parameter is very low, the convergence of RLS is not acceptable where it can be seen in Figure 7. If the iteration number is very high, the RLS convergence will not be affected countless as seen from Figure 8. The best convergence is taken when the regularization parameter is high (0.5) which is obtained from Figure 8.

As seen from the simulation results below, RLS algorithm is more successful than LMS algorithm in channel 1 because the error value is slower and its convergence is better. For the second and third channel forms in terms of same trial numbers and the same iteration numbers, LMS algorithm provides better convergence. Also the best convergence of LMS is taken from high iteration number but RLS does not require very high iteration number as seen from the Figures above. The step size parameter of LMS is 0.001 and regularization parameter of RLS is 0.005, and the other parameters are same. The joint results of the LMS and RLS are shown in Figures 9, 10 and 11

for channels 1, 2 and 3 respectively. RLS is blue line, LMS is red line. For 3 Figures the number of iterations is 4000.

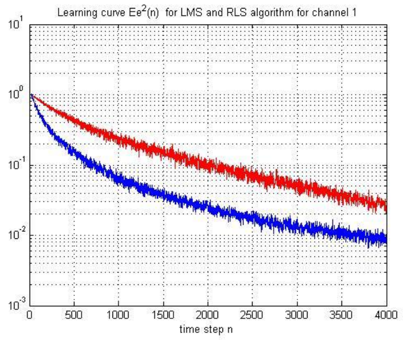

Figure 9: Joint LMS-RLS for channel 1

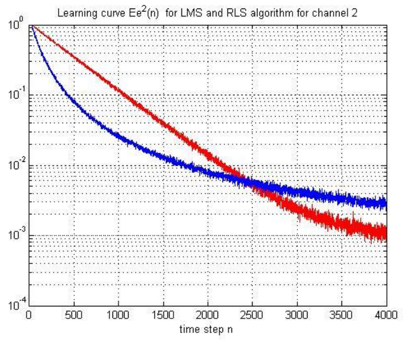

Figure 10: Joint LMS-RLS for channel 2

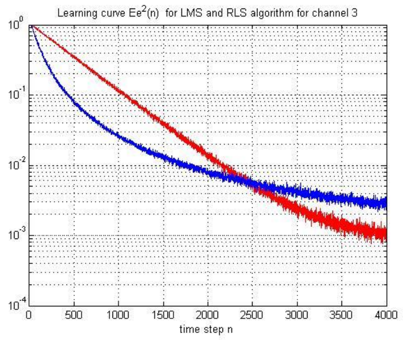

Figure 11: Joint LMS-RLS for channel 3

IV CONCLUSIONS

In this study the LMS and RLS algorithms for Adaptive Equalization for three different channel conditions is presented by using MATLAB. The simulation results of LMS and RLS algorithms are obtained for different channel forms, step-size parameters, regularization parameters and iteration numbers. The affects of these parameters are analyzed. Advantages and disadvantages of algorithms are discussed. The performance of LMS and RLS algorithms are compared in terms of three different channel conditions. Simulation results are generally shows that the RLS algorithm outperforms LMS algorithm because it provides faster and better convergence and also it seems more stable than LMS algorithm. However RLS algorithm is more complex than LMS algorithm because LMS algorithm does not include matrix inversion operations, thus RLS algorithm brings additional costs to computational operations. In addition it is also obtained from the simulation results that the step–size and regularization parameters must be chosen carefully for providing stability.

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